Analytical Hierarchy Process (AHP): A Multi-Objective Decision Making Technique

Jason C.H. Chen, Ph.D.
Professor of MIS
School of Business
Gonzaga University
Spokane, WA 99258
chen@jepson.gonzaga.edu

Analytical Hierarchy Process

• In many situations one may not be able to assign weights to the different decision factors. Therefore one must rely on a technique that will allow the estimation of the weights.
• What is a solution?
• One such process, The Analytical Hierarchy Process (AHP), involves pairwise comparisons between the various factors.

Application Case of AHP

• Jane is about to graduate from college and is trying to determine which job offer to accept. She plans to choose between three offers by determining how well each offer meets the following criteria (objectives):
  – High starting salary
  – Quality of life in city where job is located
  – Interest of work
  – Nearness of job to family

Determine the problem

• What job offer will give Jane possibly highest satisfaction?
• Structure the hierarchy by putting the top objective (satisfaction with job), criteria, and alternatives as follows.

Assumptions

• Jane has hard time in prioritizing those criteria. In other words, she needs to find one way to decide the weights for those criteria. AHP provides such a function.
The Principle of the AHP …

- The principle of the AHP relies on the pairwise comparison. This comparison is carried out using a scale from 1 to 9 as follows:
  - 1 Equally preferred
  - 2 Equally to Moderately preferred
  - 3 Moderately preferred
  - 4 Moderately to Strongly preferred
  - 5 Strongly preferred
  - 6 Strongly to Very Strongly preferred
  - 7 Very Strongly preferred
  - 8 Very to Extremely Strongly preferred
  - 9 Extremely preferred

Since \( n=4 \), there are 6 \([n*(n-1)/2]\) judgments required to develop each matrix. Why?

Using the same steps of 3 and 4 (see handout) to determine the score of each alternative on each criterion. Take the first criterion “Salary” as an example. One pairwise matrix is constructed as follows (details see step 4 on the handout):

In terms of criterion of “Salary”, Job A is moderately important ("2") than Job B. However, Job A is essentially more important ("4") than Job C.
The next two pairwise matrices (for “Life Quality” and “Interest”) are as follows (see step#6 on the handout):

**Quality**

<table>
<thead>
<tr>
<th></th>
<th>Job A</th>
<th>Job B</th>
<th>Job C</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td>1</td>
<td>1/2</td>
<td>1/3</td>
</tr>
<tr>
<td><strong>A</strong></td>
<td>Job A</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>A</strong></td>
<td>Job B</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td><strong>A</strong></td>
<td>Job C</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

**Interest**

<table>
<thead>
<tr>
<th></th>
<th>Job A</th>
<th>Job B</th>
<th>Job C</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td>1</td>
<td>1/7</td>
<td>1/3</td>
</tr>
<tr>
<td><strong>A</strong></td>
<td>Job A</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>A</strong></td>
<td>Job B</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td><strong>A</strong></td>
<td>Job C</td>
<td>3</td>
<td>1/3</td>
</tr>
</tbody>
</table>

The last pairwise matrix (for “Nearness to family”) is listed below:

<table>
<thead>
<tr>
<th></th>
<th>Job A</th>
<th>Job B</th>
<th>Job C</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td>1</td>
<td>1/4</td>
<td>1/7</td>
</tr>
<tr>
<td><strong>A</strong></td>
<td>Job A</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>A</strong></td>
<td>Job B</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td><strong>A</strong></td>
<td>Job C</td>
<td>7</td>
<td>2</td>
</tr>
</tbody>
</table>

How to verify that the data entered in the comparison matrices is acceptable

Consistency Index (C.I) is computed as follows (see handout, p.5)

\[ C.I. = \frac{\lambda_{max} - n}{n-1} = \frac{4.0474-4}{3} = 0.0159 \]

We then compare the value of C.I. to the value of random index (R.I.). If the ratio of C.I. to R.I. is less than 10%, then we can say the judgment process is relatively consistent and the matrix is acceptable. Otherwise, the decision maker may need to re-examine the judgment process and re-compare criteria or alternatives. The consistency ratio (C.R.) is computed as follows:

\[ C.R. = \frac{C.I.}{R.I.} \]

Random Indices (R.I.) for Consistency Check

| R.I. | 0.58 | 0.90 | 1.12 | 1.24 | 1.32 | 1.41 | 1.45 | 1.51 |

We will open an existing model

http://www.hipre.hut.fi
or
http://hipre.aalto.fi/

File name: mbus673.jmd
AHP - 19

Dr. Chen – Business Intelligence

AHP - 20

Dr. Chen – Business Intelligence

Result from “Analysis of Composite Priorities ...”

According to the BAR chart, AHP suggests that Jane should take Job B.

AHP - 21

Dr. Chen – Business Intelligence

Result from “Analysis of Composite Priorities ...”

According to the “Values”, AHP suggests that Jane should take Job B (you need to “Add total”, see the next slide).

AHP - 22

Dr. Chen – Business Intelligence

Value Tree

Composite Priorities

job A      job B      job C

salary     0.293      0.146      0.073

life quality 0.016      0.053      0.029

interest 0.021      0.161      0.059

nearness to family 0.144

overall 0.344      0.346      0.248

Step 7 (p.5)

Save your work again

(p.4 of Handout)